First study of $B \to \pi$ semileptonic decay form factors using NRQCD

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We present a quenched calculation of the form factors of the semileptonic weak decay $B \to \pi l \bar{\nu}$ with $O(1/m_Q)$ NRQCD heavy quark and Wilson light quark on a $16^3 \times 32$ lattice at $\beta = 5.8$. The form factors are evaluated at six heavy quark masses, in the range of $m_Q \sim 1.5 - 8$ GeV. $1/m_Q$ dependence of matrix elements are investigated and compared with HQET predictions. We observe clear signal for the form factors near q^2_{max}, even at the b-quark mass range. $f^0(q^2_{max})$ is compared with f_B/f_π based on the soft pion theorem and significant difference is observed.

1. Introduction

Lattice study of B decay matrix elements is important for the determination of Cabibbo-Kobayashi-Maskawa matrix elements, and for investigations of applicability of Heavy quark effective theory (HQET) which is extensively applied to phenomenological studies. In this work, we calculate $B \to \pi$ form factors using the heavy quark described by $O(1/m_Q)$ NRQCD and Wilson light quark [1].

2. Simulation

Numerical simulation is performed on 120 configurations of a $16^3 \times 32$ lattice at $\beta = 5.8$ in the quenched approximation. Wilson light quark is employed with three values of κ, 0.1570, 0.1585 and 0.1600, which leads $\kappa_c = 0.163461(69)$ and lattice cutoff $a^{-1} = 1.714(63)$ GeV determined from ρ meson mass. Light quark field is normalized with the factor $\sqrt{1 - \frac{3\kappa}{4\kappa_c}}$ [2].

The heavy quark is described with NRQCD including $O(1m_Q)$ corrections which leads following evolution equation.

$$G_\varphi(t{=}1) = \left(1 - \frac{1}{2n} H_0\right)^n U_4^\dagger \left(1 - \frac{1}{2n} H_0\right)^n G_\varphi(0),$$

$$G_\varphi(t{+}1) = \left(1 - \frac{1}{2n} H_0\right)^n U_4^\dagger \left(1 - \frac{1}{2n} H_0\right)^n \times (1 - \delta H)\, G_\varphi(t). \quad (1)$$

$$H_0 = -\frac{1}{2m_Q} \Delta^{(2)}, \qquad \delta H = -\frac{1}{2m_Q}\, \vec{\sigma} \cdot \vec{B}, \quad (2)$$

where $\Delta^{(2)}$ denotes the lattice Laplacian and B is the chromomagnetic field. The stabilizing parameter n should satisfy $n > 3/2m_Q$.

For the heavy quark, eight values of mass and stabilizing parameter are used: $(m_Q, n) = (5.0,1)$, $(2.6,1)$, $(2.1,1)$, $(2.1,2)$, $(1.5,2)$, $(1.2,2)$, $(1.2,3)$, and $(0.9,2)$. $m_Q = 2.6$ and 0.9 roughly correspond to the b- and c-quark masses. The mean-field improvement[3] is applied to the heavy quark evolution equation with $u_0 = \langle \frac{1}{3} U_{plaq.} \rangle^{1/4} = 0.867994(13)$.

The matrix elements are extracted from three point correlation functions,

$$C^{(3)}_\mu(p,k;t_f,t_s,t_i) = \sum_{\vec{x}_f} \sum_{\vec{x}_s} e^{-i\vec{p}\cdot\vec{x}_f}\, e^{-i(\vec{k}-\vec{p})\cdot\vec{x}_s} \times \langle O_B(x_f) V^\dagger_\mu(x_s) O_\pi(t_i,0) \rangle.$$

We use 20 rotationally nonequivalent sets of $(\vec{p}, \pi)$ with $|\vec{p}|$, $|\vec{k}| \leq \sqrt{3} \cdot 2\pi/16$. The source and the current operators are set on the time slices $t_i = 4$ and $t_s = 14$ respectively. The matrix elements are extracted in the region $t_f = 23 - 28$.

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Numerical simulations were carried out on Intel Paragon XP/S at INSAM (Institute for Numerical Simulation and Applied Mathematics) in Hiroshima University.

We estimate the effect of perturbative corrections to the heavy quark self-energy and the current [4]. In some cases, we use larger values of n for the perturbation than those in the simulation: $(m_Q, n) = (5.0, 1)$, $(2.6, 2)$, $(2.1, 2)$, $(1.5, 3)$, $(1.2, 3)$, and $(0.9, 6)$. This is because of the singularities encountered in the perturbative expressions for some set of (m_Q, n) with small n. The multiplicative part of the current renormalization constant is calculated with massless Wilson quark for vanishing external momenta. Two scales $q^* = \pi/a$ and $1/a$ are considered to define the expansion parameter g_V^2.

3. Results

It is useful to define following quantity.

$$\hat{V}_\mu(\vec{p}, \vec{k}\,) = \frac{\langle\, \pi\,(\,\vec{k}\,)\,|\,V_\mu\,|\,B\,(\,\vec{p}\,)\,\rangle}{\sqrt{2\,E_\pi(k)}\,\sqrt{2\,E_B(p)}} \tag{3}$$

This expression can be entirely composed of numerical results, without any assumption such as a dispersion relation. It is also convenient for a comparison with HQET predictions. According the heavy quark symmetry, for $\vec{p} = 0$, $\hat{V}_\mu$ takes constant value in the leading order of $1/m_Q$:

$$\hat{V}_4(\vec{p}=0, \vec{k}) \;=\; \hat{V}_4^{(0)}\left[1 + c_4^{(1)}/m_B + \cdots\right], \tag{4}$$

$$\begin{aligned}\hat{V}_k(\vec{p}=0, \vec{k}) \;&\equiv\; \vec{\hat{k}} \cdot \vec{\hat{V}}(\vec{p}{=}0, \vec{k})/\vec{\hat{k}}^{2} \\ &=\; \hat{V}_k^{(0)}\left[1 + c_k^{(1)}/m_B + \cdots\right],\end{aligned} \tag{5}$$

where $\hat{k}_i = 2\sin(k_i/2)$. Upper two of Figure 1 show the results of $\hat{V}_4$ for $\vec{p} = \vec{k} = 0$ and $\hat{V}_k$ for $|\vec{k}| = 2\pi/16$ in the case of $\kappa = 0.1570$. They are evaluated at three renormalization scales, mean-field tree, $q^* = \pi/a$ and $1/a$. Both $\hat{V}_4$ and $\hat{V}_k$ less depend on m_B in comparison with f_B case [5]. The spacial component of $\hat{V}$ is more affected by the perturbative corrections than the temporal one is. It is also predicted that

$$\begin{aligned}\hat{V}_p(\vec{p}=0, \vec{k}) \;&\equiv\; \lim_{\vec{p}^{\,2}\to 0} \vec{\hat{p}} \cdot \vec{\hat{V}}(\vec{p}, \vec{k})/\,\vec{\hat{p}}^{\,2} \\ &=\frac{1}{m_B}\hat{V}_p^{\prime(0)}\left[1 + c_p^{(1)}/m_B + \cdots\right].\end{aligned} \tag{6}$$

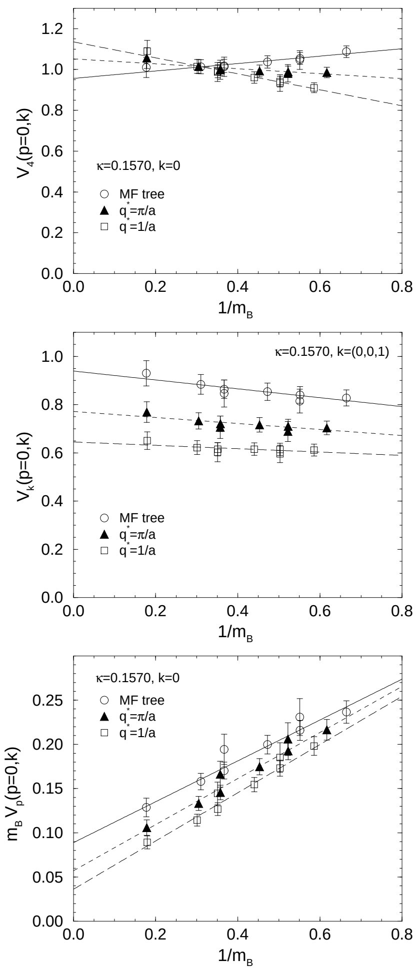

Figure 1. m_B dependence of $\hat{V}$ for $\kappa = 0.1570$. $\hat{V}_4$ (top figure), $\hat{V}_k$ (middle), and $m_B \hat{V}_p$ (bottom) with three renormalization scales for each.

We extrapolate $\hat{V}_p$ at finite $\vec{p}$ to $\vec{p} = 0$ linearly in $\vec{p}^{\,2}$ to determine $\hat{V}_p(\vec{p}{=}0, \vec{k})$. $\hat{V}_p(\vec{p} = 0, \vec{k} = 0)$ multiplied by m_B is also displayed in Figure 1. Contrary to the cases of $\hat{V}_4$ and $\hat{V}_k$, $O(1/m_B)$ effect is significant for $\hat{V}_p$.

The matrix elements are expressed in terms of two form factors, f^0 and f^+ :

$$\langle\pi(k)|V_\mu|B(p)\rangle = \left(p + k - q\frac{m_B^2 - m_\pi^2}{q^2}\right)_\mu f^+(q^2)$$

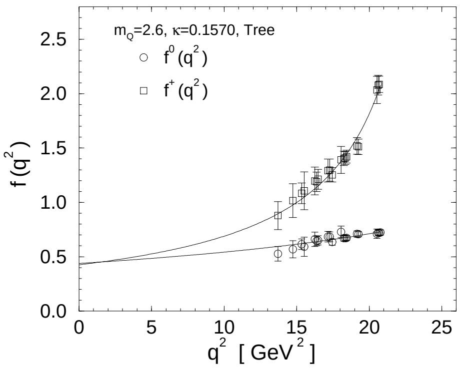

Figure 2. Form factors for $m_Q = 2.6$ and $\kappa = 0.1570$ in the mean-field tree level. Solid lines correspond to the results of single pole fit.

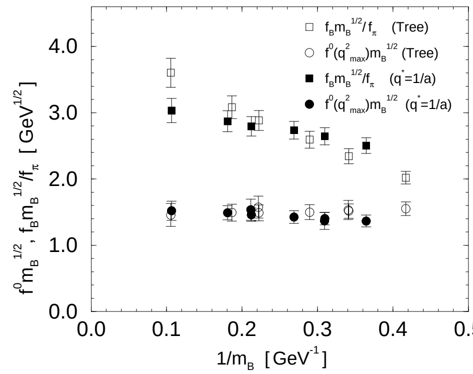

Figure 3. Comparison of $f^0(q^2_{max})$ with f_B/f_π in two scales, mean-field tree and $q^* = 1/a$.

$$+q_\mu \frac{m_B^2 - m_\pi^2}{q^2} f^0(q^2). \qquad (7)$$

To obtain the form factors from $\hat{V}_\mu$, it is necessary to assume some dispersion relations for π and B. For both of B and π, we employ the form $E(p)^2 = m^2 + \sum_i 4\sin^2(p_i/2)$ to assure the on-shell conditions. Figure 2 shows the obtained form factors for $m_Q = 2.6$, $\kappa = 0.1570$, in the mean-field tree level. It is observed that for larger m_Q, f^+ increase rapidly near q^2_{max}, which may be explained as the B^* pole contribution. We fit the form factors to a single pole function, whose result is displayed in Figure 2 as the solid lines. The fit of f^+ gives the exchanged vector particle mass $m_{pole} = 2.979(49)$ which is comparable with $m_{B^*} = 3.2502(62)$ determined from the two point correlator. The renormalization corrections change the shape of f^+ more than f^0, because of asymmetric effect on temporal and spacial components. Even after the renormalization is incorporated, the pole fit gives similar m_{pole}.

We extrapolate the matrix elements linearly in $1/\kappa$ to the chiral limit. Alternative extrapolation using the form factors gives 7 % discrepancy of $f^0(q^2_{max})$ for $m_Q = 2.6$ and 14 % for $m_Q = 0.9$. At the zero recoil point, the prediction of heavy meson effective theory [6] advocates the linear extrapolation of the matrix element. The results at κ_c are much noisy for quantitative discussions.

Finally, we consider the implication of soft pion theorem [7,6]. For the massless pion limit, $f^0(q^2_{max})$ should equal to f_B/f_π. This relation is examined in Figure 3, using our result on f_B determined with slightly different form of NRQCD [5]. Significant difference is observed in large m_B region. Similar result is obtained in the work using Fermilab action for the heavy quark [8]. The origin and physical meaning of this discrepancy remains as a future problem.

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